Algebraic Proof

Question Paper

Course	Edexcel IGCSE Maths
Section	2. Equations, Formulae & Identities
Topic	Algebraic Proof
Difficulty	Very Hard

Time allowed: 60

Score: /44

Percentage: /100

Question 1

(i)
Factorise $2t^2 + 5t + 2$.

[2]

(ii)
t is a positive whole number.
The expression $2t^2 + 5t + 2$ can never have a value that is a prime number.
Explain why.

[1]

[3 marks]

Question 2

n is an integer.

Prove algebraically that the sum of $\frac{1}{2}\, n(n + 1)$ and $\frac{1}{2}\,(n + 1)(n + 2)$ is always a square number.

[2 marks]

Question 3

Here are the first five terms of an arithmetic sequence.

7 13 19 25 31

Prove that the difference between the squares of any two terms of the sequence is always a multiple of 24.

[6 marks]

Question 4

Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

[2 marks]

Question 5

The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

[3 marks]

Question 6

Prove that when the sum of the squares of any two consecutive odd numbers is divided by 8, the remainder is always 2
Show clear algebraic working.

[3 marks]

Question 7

Using algebra, prove that, given any 3 consecutive whole numbers, the sum of the square of the smallest number and the square of the largest number is always 2 more than twice the square of the middle number.

[3 marks]

Question 8

Using algebra, prove that, given any 3 consecutive even numbers, the difference between the square of the largest number and the square of the smallest number is always 8 times the middle number.

[3 marks]

Question 9a

Here are the first four terms of a sequence of fractions.

$$\frac{1}{1} \quad \frac{2}{3} \quad \frac{3}{5} \quad \frac{4}{7}$$

The numerators of the fractions form the sequence of whole numbers $1\ 2\ 3\ 4\ \ldots$
The denominators of the fractions form the sequence of odd numbers $1\ 3\ 5\ 7\ \ldots$

Write down an expression, in terms of n, for the nth term of this sequence of fractions.

[2 marks]

Question 9b

Using algebra, prove that when the square of any odd number is divided by 4 the remainder is 1

[3 marks]

Question 10

The table gives information about the first six terms of a sequence of numbers.

Term number	1	2	3	4	5	6
Term of sequence	$\frac{1 \times 2}{2}$	$\frac{2 \times 3}{2}$	$\frac{3 \times 4}{2}$	$\frac{4 \times 5}{2}$	$\frac{5 \times 6}{2}$	$\frac{6 \times 7}{2}$

Prove algebraically that the sum of any two consecutive terms of this sequence is always a square number.

[4 marks]

Question 11

Prove that $x^2 + x + 1$ is always positive.

[3 marks]

Question 12a

The diagram shows a cross placed on a number grid.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

L is the product of the left and right numbers of the cross.
T is the product of the top and bottom numbers of the cross.
M is the middle number of the cross.

Show that when $M = 35$, $L - T = 99$.

[2 marks]

Question 12b

Prove that, for any position of the cross on the number grid above, $L - T = 99$.

[5 marks]